Alexey Shevyakov: Conservation laws, similarity reductions, and exact solutions for helically symmetric incompressible fluid flows

Submitted by Jorge Cisneros on May 7, 2020 - 6:28pm

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Date: May 14, 2020

Title: Conservation laws, similarity reductions, and exact solutions for helically symmetric incompressible fluid flows

Abstract: For partial differential equations in three spatial dimensions, the helically invariant reduction arises when a model depends on a reduced set of independent variables $(t, r, \xi)$, where $\xi = az + b\varphi$, emerging from a cylindrical coordinate system $(r, \varphi, z)$. First, we present full sets of helically invariant viscous and inviscid time-dependent fluid flow equations. Special cases of rotationally symmetric and plane flows are obtained as limiting cases of helical invariance. Second, local conservation laws of helically invariant systems are systematically derived through the direct construction method. Multiple new sets of conservation laws for both inviscid and viscous flows, including families that involve arbitrary functions, are derived. For both Euler and Navier-Stokes flows, infinite sets of vorticity-related conservation laws are obtained. In particular, for Euler flows, we obtain a family of conserved quantities that generalize helicity. Additional conservation laws are found for two-component flows. Third, we derive some closed-form exact solutions for the helically invariant Navier-Stokes equations. The approach is based on an invariant solution ansatz emerging from the Galilean group in helical coordinates, which leads to a new equation for the radial velocity component in the helical frame. Further, an exact linearization of the Navier-Stokes equations in terms of Beltrami flows is used to derive exponentially decaying time-dependent separated solutions, involving confluent Heun-type functions in radial direction. This work was done in collaboration of the group of Prof. M. Oberlack (TU Darmstadt, Germany).

YouTube: https://youtu.be/8bxQWi0MnmE
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